

I. Observations

- A. Make a bunch of observations with polarizers. Use whatever layers or combinations of layers you want to look at whatever you like. Record your observations here.

- B. You probably have some ideas about how to account for these observations. If you feel like it, write them down here; this can help you reflect on your explanations as they develop.

II. Physical model

A quantum (plural: quanta) is something that can be counted. For example, in a cup of rice, each grain of rice is a quantum. In physics, a quantum is never subdivided.

Your instructor will demonstrate a physical representation of a quantum model of light. The model may be summarized as follows:

- i. **Polarization is orientation:** Each photon is oriented (“polarized”) in a direction that matters for its interaction with a polarizer.
- ii. **Transmission is probabilistic:** Each photon is transmitted or blocked by the polarizer probabilistically depending on its orientation. An aligned photon is transmitted, a perpendicular photon is blocked, and a photon at a 45-degree angle has a 50-50 chance of being transmitted or blocked.

- A. Take some time to play with the physical representation demonstrated by the instructor and work out for yourself how it corresponds to the model of light described above. Make notes here.
- B. Try to explain the observations you made above using this physical model. (You don’t have to go through all of them.) To test yourself, use the model to explain the following observations:
 - a. Two perpendicular polarizers block light.
 - b. A third polarizer added between perpendicular polarizers can cause some light to be transmitted through all three of them.
 - c. Some screens appear dark purple when viewed through a polarizer held in a particular orientation.
- C. Based on your observations, is there anything you want to challenge, add, rephrase, or clarify about the model of light described above?

- D. Two students are discussing what they think happens to photons that are transmitted through the polarizer.

Sam: “The photon isn’t changed by the polarizer. Why would it be? It is what it is.”

Skye: “I’m thinking about the case of perpendicular polarizers. If a photon is initially at a 45-degree angle to the first polarizer, it would have a 50-50 chance of transmission through both the first and second polarizers, right? But I saw that essentially nothing was transmitted. Something isn’t right.”

What change do you suggest to resolve Skye’s concern?

- E. If you want to, go back to your notes about your initial ideas of how polarizers work and compare to (or contrast with) this representation.

★ *There will be a whole class discussion after this section.*

III. Mathematical model

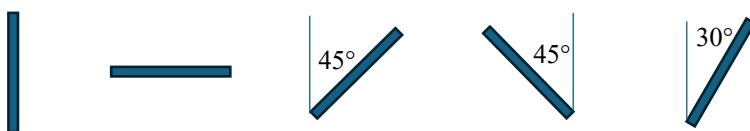
In physics, the “state” of an object or system is a description of the object or system at a given time, including whatever properties are relevant to predicting its behavior.

A. Polarization states

When predicting observations with a polarizer, the most important property is the *polarization state* of the photons involved, denoted with an angle in this special format: $|\theta\rangle$. The reference angle is our choice. For example, we might choose:

Vertical polarization	$ 0^\circ\rangle$
Horizontal polarization	$ 90^\circ\rangle$
Polarization 45° right of vertical	$ 45^\circ\rangle$
Polarization 45° left of vertical	$ -45^\circ\rangle$

- Five photons are represented below as if viewed edge-on. Write the polarization state of each one using the above notation.

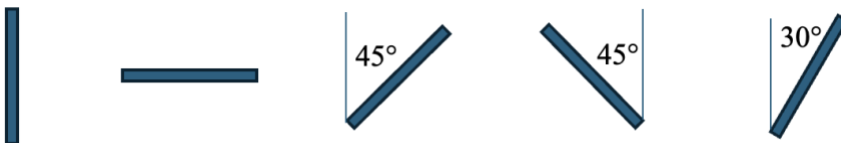


2. Each of the above photons is incident on a vertical polarizer (that is, photons with a vertical polarization are transmitted). For each photon shown above:
 - a. Calculate the probability that it will be transmitted. (If you can't calculate one of them, state whether the probability would be greater than, less than, or equal to 50%.)
 - b. In the case that the photon is transmitted, write its polarization state after transmission (in $|\theta\rangle$ notation).
3. Polarization states that are perfectly aligned with (or perpendicular to) a given polarizer are associated with a definite outcome: the photon will definitely be transmitted (or blocked). We will call these polarization states “definite states” for that polarizer.
 - a. How could you orient a polarizer such that a photon polarization state of $|30^\circ\rangle$ is a definite state for that polarizer? (Hint: There are at least two orientations and maybe four, depending on how you count.)
 - b. Describe a way to prepare a bunch of photons in the polarization state $|30^\circ\rangle$.
 - c. Explain why a photon polarization state that is a definite state for one polarizer might not be a definite state for a differently oriented polarizer.

B. States as vectors

A polarization state is a lot like a vector. In fact, we can represent it as a unit vector in three-dimensional space. We will choose the direction of this unit vector to be perpendicular to the photon's direction of travel, at the angle denoted by the polarization state.

1. Draw a vector representing the polarization state of each of the photons pictured.



Any vector can be expressed as the sum of other vectors; this is called “vector superposition.” In just the same way, a “superposition state” for a given polarizer is a sum of definite states for that polarizer. Superposition can be helpful for predicting the transmission of photons through a polarizer.

Vector superposition

2. In the box at right, draw a vector addition picture representing the $|45^\circ\rangle$ unit vector as a sum of vectors in the $|0^\circ\rangle$ and $|90^\circ\rangle$ directions. Determine the lengths of the vectors in the $|0^\circ\rangle$ and $|90^\circ\rangle$ directions.
3. Write an expression for the $|45^\circ\rangle$ unit vector in terms of vectors in the $|0^\circ\rangle$ and $|90^\circ\rangle$ directions.
4. Explain how to use this expression to predict the probability of transmission of a photon in the $|45^\circ\rangle$ state through a vertically oriented polarizer.
5. Repeat the above procedure to write the $|30^\circ\rangle$ unit vector in terms of vectors in the $|0^\circ\rangle$ and $|90^\circ\rangle$, and use that expression to predict the probability of transmission through a vertically-oriented polarizer.
6. Propose a general rule or procedure for predicting the probability of transmission of a photon in a given polarization state through a vertical polarizer.
7. Suppose you have a photon in a polarization state $|\theta\rangle = a|0^\circ\rangle + b|90^\circ\rangle$. What is the probability of this photon being transmitted through a vertical polarizer? A horizontal polarizer?
8. (Bonus question) Describe a way to prepare a bunch of photons in the polarization state $|\theta\rangle = a|0^\circ\rangle + b|90^\circ\rangle$.

Vector superposition

IV. Three polarizers

Let's revisit the case of the three polarizers using this new mathematical formalism of polarization states. Recall that while perpendicular polarizers block light, a third polarizer added diagonally between them causes some light to be transmitted. Assume the first polarizer is vertical, so that any photon transmitted through it has polarization state $|\theta_1\rangle = |0^\circ\rangle$. The second polarizer is oriented 45° clockwise from the first one, and the third polarizer is oriented 45° clockwise from the second one.

1. What are the definite photon polarization states associated with the following? (There are two each.)
 - a. the first polarizer?
 - b. the second polarizer?
 - c. the third polarizer?

2. After being transmitted through the first polarizer, a photon is in the state $|\theta_1\rangle = |0^\circ\rangle$. To predict what's transmitted through the second polarizer, rewrite $|\theta_1\rangle$ as a superposition of the definite states of the second polarizer. (In other words, draw a triangle showing vectors pointing in the 45° and -45° directions adding up to a unit vector in the vertical direction; find the lengths of the sides of the triangle; and write an algebraic expression.)

Vector superposition

- a. What is the probability that a photon in the $|\theta_1\rangle$ state is transmitted through the second polarizer? Explain how you can tell from the superposition state.

- b. In the case that the photon is transmitted, what is its polarization state after transmission?

3. Now write $|\theta_2\rangle$ as a superposition of the definite states of the *third* polarizer. (*Hint*: Draw a triangle whose hypotenuse is $|\theta_2\rangle$.)

What is the probability that a photon in the $|\theta_2\rangle$ state is transmitted through the third polarizer? Explain how you can tell from the superposition state.

Vector superposition

4. If a million photons are incident on the first polarizer, about how many will be transmitted through the second and third polarizers?

V. Reflection

A. Look back over your notes for this tutorial. What are the most important conclusions or findings from your work? Summarize them here in a couple sentences.

B. Are you concerned about any of the conclusions you have reached so far? What are your concerns?

C. What did you feel most prominently when working on this tutorial, including both physical sensations and emotions?

★ *There will be a whole class discussion after this section.*